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ON THE APPLICATION OF THE SPLITTING METHOD FOR NUMERICAL CALCULATION OF HEAT-CONDUCTING GAS FLOWS IN CURVILINEAR COORDINATES

N. N. Yanenko, V. D. Frolov, V. Ye. Neuvazhayev

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16. Abstract

Eulerian, Langrangian and implicit difference methods of describing motion are juxtaposed and the latter is employed to resolve the two-dimensional problem of axially symmetric motion with thermal conductivity in heterogeneous media. Computer results are presented.

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ON THE APPLICATION OF THE SPLITTING METHOD FOR NUMERICAL CALCULATION OF HEAT-CONDUCTING GAS FLOWS IN CURVILINEAR COORDINATES

N. N. Yanenko, V. D. Frolov, V. Ye. Neuvazhayev[1]

An algorithm of the numerical calculation for the problem of axially symmetric motion with consideration of thermal conductivity in heterogeneous media, based on the application of the splitting (fractional step) method [1, 4, 5], is presented in this article. The solution of the two-dimensional problem is reduced to a series of unidimensional calculations. /74*

One of the most important aspects of two-dimensional problems is the choice of the method of describing motion. The Euler and Lagrange methods are known extensively. In the Euler method, the distribution of parameters of the state of a medium in motion is determined for each given point in space. This method is convenient for numerical calculations in the presence of fixed boundaries and absence of medium interfaces, and permits large deformations of matter. In the Lagrange method, motion and state are determined for each fixed particle of matter. In numerical calculations, this method is convenient in the presence of interfaces, but it does not tolerate large displacements of particles relative to each other, for example sliding of layers.

Combined methods have appeared recently, incorporating the advantages of both concepts for numerical calculations. Thus, a mobile Euler network is used in [2]. V. F. D'yachenko, as the authors understand, used in calculations Euler-Lagrangian coordinates, in which the interfaces of the media are Lagrangian lines.

Also used in the cited work is a combined Euler-Lagrange method of describing motion, when one set of coordinate lines, coinciding with the interfaces, is Lagrangian and the other is Eulerian. It is then possible to trace the interfaces and to calculate the flow of matter in the layers. In view of the fact that the shape of interfaces may vary in time, they are connected

*Numbers in the margin indicate pagination in the foreign text.

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to a new curvilinear coordinate system at each step of the calculation in order to maintain the boundaries as coordinate lines.

Implicit difference methods, permitting calculation with a sufficiently large step in time, are used in this work.

In § 1 the equations describing the motion of a continuum are written in arbitrary curvilinear coordinates. This makes it possible to use an arbitrary metric, and on the boundaries coinciding with the Lagrangian set of coordinate lines, to maintain the corresponding contravariant component of the velocity /75 vector continuously, whereupon it is not especially necessary to separate the interfaces during the calculation.

Further, in § 2 these equations are split into two systems, in each of which are considered only derivatives of one direction. In the system containing the derivatives on the coordinate corresponding to the interfaces, we convert to a Lagrangian mass coordinate. For each of the systems obtained in § 3 we write implicit difference systems, which serve as a foundation for the successive dispersion method [5]. We first solve the system that describes motion in the Euler network, and then we use the values obtained and calculate the system in Lagrangian coordinates, i.e., the solution of the problem is reduced to calculation of M one-dimensional problems in the Eulerian network and K one-dimensional problems in the Lagrangian network, where M and K are the numbers of counting intervals on each coordinate set.

Some results of calculations, done by the program that executes the described method in the computer, are presented in § 4. The program was written by V. M. Gribov, V. I. Legon'kov and L. N. Khokhryakova. A. I. Zuyev also took part in the development of the method. The authors express their gratitude to all coworkers who assisted them in this work.

§ 1. Basic Equations. The motion of heat-conducting gas is described by the following equations:

$$\frac{d\rho}{dt} + \rho \operatorname{div} \vec{v} = 0, \quad \rho \frac{d\vec{v}}{dt} + \operatorname{grad} p = 0,$$
$$\frac{d\varepsilon}{dt} + p \frac{d\frac{1}{\rho}}{dt} = \frac{1}{\rho} \operatorname{div} (\varkappa \operatorname{grad} T). \tag{1.1}$$

Here $p = p(\rho, T)$ is pressure, $\varepsilon = \varepsilon(\rho, T)$ is internal energy, ρ is density, T is temperature, $\vec{v}$ is the velocity vector, $\kappa = \kappa(\rho, T)$ is the coefficient of thermal conductivity.

Let y_k ($k = 1, 2, 3$) be Cartesian orthogonal coordinates, x^i ($i = 1, 2, 3$) be arbitrary curvilinear coordinates. We will introduce the definitions:

$$
\begin{aligned}
& g_{ik} = \frac{\partial y_\alpha}{\partial x^i} \frac{\partial y_\alpha}{\partial x^k}; \quad g = \begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{vmatrix}; \\
& g^{ik} = \frac{\text{algebraic component}}{g}; \\
& \Gamma^k_{\lambda i} = \frac{1}{2} g^{sk} \left(\frac{\partial g_{s\lambda}}{\partial x^i} + \frac{\partial g_{is}}{\partial x^\lambda} - \frac{\partial g_{i\lambda}}{\partial x^s} \right); \\
& \nabla_i v^k = \frac{\partial v^k}{\partial x^i} + v^\lambda \Gamma^k_{\lambda i}.
\end{aligned}
\tag{1.2}
$$

We will choose the curvilinear coordinate system at each moment of time t so that the lines $x^1 = r = \text{const}$ coincide with the interfaces of the layers, and also so that $x^2 = \theta$ and $x^3 = \phi$, where θ and ϕ are the polar distance and length, respectively. Let $R = R(r, \theta)$ be the distance between the given point with coordinates r, θ and the origin of the coordinates. Then Cartesian coordinates y_k and curvilinear coordinates x^i will be connected by the relations /76

$$
\begin{aligned}
y_1 &= R \sin\theta \cos\varphi; \\
y_2 &= R \sin\theta \sin\varphi; \\
y_3 &= R \cos\theta.
\end{aligned}
\tag{1.3}
$$

Expressions (1.2), with consideration of (1.3), acquire the form

$$
\begin{aligned}
& g_{11} = R_r^2; \quad g_{12} = R_r R_\theta; \quad g_{13} = 0, \\
& g_{21} = g_{12}; \quad g_{22} = R^2 + R_\theta^2; \quad g_{23} = 0; \\
& g_{31} = 0; \quad g_{32} = 0; \quad g_{33} = R^2 \sin^2\theta; \\
& g = R^4 R_r^2 \sin^2\theta;
\end{aligned}
\tag{1.4}
$$

$$\Gamma'_{11} = \frac{R_{rr}}{K_r}; \quad \Gamma^1_{12} = \frac{RR_{r\theta} - R_r R_\theta}{RR_\theta}; \quad \Gamma^1_{22} = \frac{RR_{\theta\theta} - 2R_\theta^2 - R^2}{RR_r};$$

$$\Gamma^2_{11} = 0; \quad \Gamma^2_{12} = \frac{R_r}{R}; \quad \Gamma^2_{22} = \frac{2R_\theta}{R}.$$

Here the subscripts r and θ denote differentiation in terms of r and θ, respectively.

If we denote

$$u = v^1; \quad v = v^2;$$

$$\frac{\partial}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r} + v\frac{\partial}{\partial \theta} \tag{1.5}$$

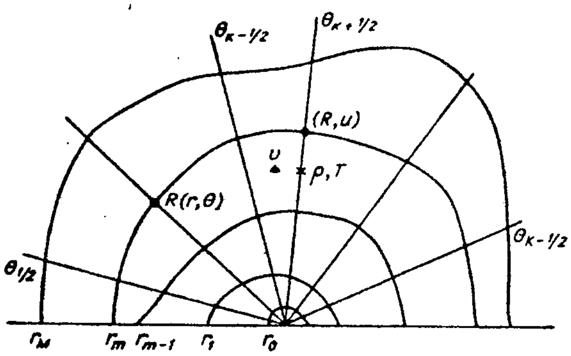
and apply transform (1.3) to equation system (1.1), we obtain

$$\begin{aligned}
&\frac{D\rho}{Dt} + \frac{\rho}{\sqrt{g}}\left(\frac{\partial\sqrt{g}\,u}{\partial r} + \frac{\partial\sqrt{g}\,v}{\partial\theta}\right) = 0;\\
&\frac{Du}{Dt} + \Gamma'_{11}u^2 + 2\Gamma'_{12}uv + \Gamma'_{22}v^2 = \frac{1}{\rho R^2 R_2^2}\left(-g_{22}\frac{\partial p}{\partial r} + g_{12}\frac{\partial p}{\partial\theta}\right);\\
&\frac{Dv}{Dt} + 2\Gamma^2_{22}uv + \Gamma^2_{22}v^2 = \frac{1}{\rho R^2 R_2^2}\left(g_{12}\frac{\partial p}{\partial r} - g_{11}\frac{\partial p}{\partial\theta}\right);\\
&\frac{D\varepsilon}{Dt} + \frac{p}{\rho\sqrt{g}}\left(\frac{\partial\sqrt{g}\,u}{\partial r} + \frac{\partial\sqrt{g}\,v}{\partial\theta}\right) = \frac{1}{\rho\sqrt{g}}\frac{\partial}{\partial r}\times\\
&\quad\times\frac{\varkappa\sqrt{g}}{R^2 R_2}\left(g_{22}\frac{\partial T}{\partial r} - g_{12}\frac{\partial T}{\partial\theta}\right) + \frac{1}{\rho\sqrt{g}}\frac{\partial}{\partial\theta}\times\\
&\quad\times\frac{\varkappa\sqrt{g}}{R^2 R_2^2}\left(-g_{12}\frac{\partial T}{\partial r} + g_{11}\frac{\partial T}{\partial\theta}\right).
\end{aligned} \tag{1.6}$$

/77

The problem is stated as follows: determine the solution of system (1.6) in some region D (see the figure), bounded by the axis of symmetry and some curve

$$r = r_B, \quad 0 \leqslant \theta \leqslant \pi, \tag{1.7}$$



if at moment $t = t_0$, $u(r, \theta, t_0)$, $v(r, \theta, t_0)$, $\rho(r, \theta, t_0)$, $T(r, \theta, t_0)$ are known and on boundary (1.7), for example, the following conditions are prescribed:

$$p = p\ (\theta,\ t); \quad \varkappa \frac{\partial T}{\partial n} = f\ (T), \tag{1.8}$$

where $\frac{\partial}{\partial n}$ denotes differentiation on the normal to curve (1.7). We will note that the flow in (1.8) can be written in coordinates r and θ:

$$\frac{\varkappa}{R R_r \sqrt{g_{22}}} \left(g_{22} \frac{\partial T}{\partial r} - g_{12} \frac{\partial T}{\partial \theta} \right) = f\ (T). \tag{1.9}$$

On the axes of symmetry $\theta = 0$; $\theta = \pi$:

$$\frac{\partial \rho}{\partial \theta} = \frac{\partial T}{\partial \theta} = \frac{\partial u}{\partial \theta} = 0; \quad v = 0.$$

§ 2. Splitting of system (1.6). We separate system (1.6) into two, if possible considering in each one of them derivatives only in terms of one of the directions r and θ:

$$\begin{aligned}
&\frac{1}{2} \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial \theta} + \Gamma'_{12} u v + \Gamma'_{22} v^2 - \frac{R \theta}{\rho R^2 R_r} \frac{\partial p}{\partial \theta} = 0; \\
&\frac{1}{2} \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial \theta} + \Gamma^2_{12} u v + \Gamma^2_{22} v^2 + \frac{1}{\rho R^2} \frac{\partial p}{\partial \theta} = 0; \\
&\frac{1}{2} \frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial \theta} + \frac{\rho}{\sqrt{g}} \frac{\partial \sqrt{g}\, v}{\partial \theta} = 0.
\end{aligned} \tag{2.1}$$

$$\frac{1}{2}\frac{\partial \varepsilon}{\partial t} + v\frac{\partial \varepsilon}{\partial \theta} + \frac{p}{\rho\sqrt{g}}\frac{\partial \sqrt{g}\,v}{\partial\theta} = \frac{1}{\rho\sqrt{g}}\frac{\partial}{\partial\theta}\times$$
$$\times\frac{\varkappa\sqrt{g}}{R^2R_r^2}\left(-R_rR_\theta\frac{\partial T}{\partial r} - R_r^2\frac{\partial T}{\partial\theta}\right);$$

$$\frac{1}{2}\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \Gamma^1_{11}u^2 + \Gamma^1_{12}uv + \frac{R^2+R_\theta^2}{\rho R^2R_r}\frac{\partial p}{\partial r} = 0;$$
$$\frac{1}{2}\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \Gamma^2_{12}uv - \frac{R_\theta}{\rho R^2R_r}\frac{\partial p}{\partial r} = 0; \tag{2.2}$$
$$\frac{1}{2}\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \frac{\rho}{\sqrt{g}}\frac{\partial\sqrt{g}\,u}{\partial r} = 0;$$
$$\frac{1}{2}\frac{\partial \varepsilon}{\partial t} + u\frac{\partial \varepsilon}{\partial r} + \frac{p}{\rho\sqrt{g}}\frac{\partial\sqrt{g}\,u}{\partial r} =$$
$$= \frac{1}{\rho\sqrt{g}}\frac{\partial}{\partial r}\frac{\varkappa\sqrt{g}}{R^2R_r^2}\left[\left(R^2+R_\theta^2\right)\frac{\partial T}{\partial r} - R_rR_\theta\frac{\partial T}{\partial\theta}\right].$$

We substitute

$$U = uR_r,\quad H = \frac{R_\theta}{R} \tag{2.3}$$

and convert in system (2.2) to the Lagrangian mass coordinate

$$dq = \rho R^2R_r\,dr.$$

Then we have

$$\frac{1}{2}\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial\theta} + 2\frac{Uv}{R} + 2Hv^2 + \frac{1}{\rho R^2}\frac{\partial p}{\partial\theta} = 0;$$
$$\frac{1}{2}\frac{\partial U}{\partial t} + v\frac{\partial U}{\partial\theta} - HUv + R\left(\frac{\partial H}{\partial\theta} - H^2 - 1\right)v^2 - \frac{H}{\rho R}\frac{\partial p}{\partial\theta} = 0; \tag{2.4}$$

$$\frac{1}{2}\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial\theta} + \frac{\rho}{\sqrt{g}}\frac{\partial}{\partial\theta}\left(\sqrt{g}\,v\right) = 0;$$
$$\frac{1}{2}\frac{\partial \varepsilon}{\partial t} + v\frac{\partial \varepsilon}{\partial\theta} + \frac{p}{\rho\sqrt{g}}\frac{\partial}{\partial\theta}\left(\sqrt{g}\,v\right) =$$
$$= \frac{1}{\rho\sqrt{g}}\frac{\partial}{\partial\theta}\frac{\varkappa\sqrt{g}}{R^2R_r}\left(-R_\theta\frac{\partial T}{\partial r} + P_2\frac{\partial T}{\partial\theta}\right); \tag{2.5}$$

$$\frac{1}{2}\frac{\partial U}{\partial t}+R\frac{\partial H}{\partial R}Uv+R^2(1+H^2)\frac{\partial p}{\partial q}=0;$$
$$\frac{1}{2}\frac{\partial v}{\partial t}-R^2H\frac{\partial p}{\partial q}=0;\quad \frac{1}{2}\frac{\partial R}{\partial t}=U; \tag{2.5}$$

/79

$$\frac{1}{2}\frac{\partial\frac{1}{\rho}}{\partial t}-\frac{\partial R^2U}{\partial q}=0;$$
$$\frac{1}{2}\frac{\partial\varepsilon}{\partial t}+p\frac{\partial R^2U}{\partial q}=\frac{\partial}{\partial q}\varkappa R^2\left[R^2(1+H^2)\rho\frac{\partial T}{\partial q}-\frac{H}{R}\frac{\partial T}{\partial\theta}\right].$$

§ 3. Calculation equations. The three-dimensional network is constructed with the aid of rays $\theta_{k+\frac{1}{2}}$ = const, constructed from some center, and lines r_m = const, where the latter are selected such that the boundaries are coordinate lines (see the figure). R, U, v, ρ, T are defined at the following points:

$$R(r_m,\theta_{k+1/2})=R_{m,k+1/2};\quad U(r_m,\theta_{k+1/2})=U_{m,k+1/2};$$
$$v(r_{m+1/2},\theta_k)=v_{m+1/2,k};$$
$$\rho(r_{m+1/2},\theta_{k+1/2})=\rho_{m+1/2,k+1/2},\quad T(r_{m+1/2},\theta_{k+1/2})=T_{m+1/2,k+1/2}.$$

If these values must be known at other points, they are determined by interpolation.

The time stage is calculated in two stages: in the first stage (2.1) is approximated by some difference system on interval $[h\tau, (n+\frac{1}{2})\tau]$ and the values U, v, ρ, T are determined; in the second stage of the system (2.2) is approximated on the interval $[(h+\frac{1}{2})\tau, h+1]$ by system (5) and the final values of U, v, ρ, T are determined for $t=t^{n+1}=(h+1)\tau$.

The difference equations are presented below. System (2.1) is written as follows:

$$W^{\pm} = \frac{v \pm |v|}{2}; \quad \Delta\theta_k = \theta_{k+1/2} - \theta_{k-1/2}; \quad \Delta\theta_{k-1/2} = \theta_k - \theta_{k-1};$$

$$\rho^{n+1/2}_{k+1/2} + \frac{\tau W^{n+}_{k+1/2}}{\Delta\theta_k}\left(\rho^{n+1/2}_{k+1/2} - \rho^{n+1/2}_{k-1/2}\right) + \frac{\tau W^{n-}}{\Delta\theta_{k+1}}\left(\rho^{n+1/2}_{k+3/2} - \rho^{n+1/2}_{k-1/2}\right) +$$

$$+ \frac{\tau \rho^n_{k+1/2}}{\sqrt{g^n_{k+1/2}}\,\Delta\theta_{k+1/2}}\left(\sqrt{g^n_{k+1}}\, v^{n+1/2}_{k-1} - \sqrt{g^n_k}\, v^{n+1/2}_k\right) = \rho^n_{k+1/2}; \qquad (3.1)$$

$$v^{n+1/2}_k + \frac{\tau W^{n+}_k}{\Delta\theta_{k-1/2}}\left(v^{n+1/2}_k - v^{n+1/2}_{k-1}\right) + \frac{\tau W^{n-}_k}{\Delta\theta_{k+1/2}}\left(v^{n+1/2}_{k+1} - v^{n+1/2}_k\right) +$$

$$+ 2\tau\left(\frac{Uv}{R}\right)^n_k + 2\tau\,(Hv^2)^n_k + \frac{\tau}{(\rho R^2)^n_k\,\Delta\theta_k}\left(\bar{p}^{n+1/2}_{k+1/2} - \bar{p}^{n+1/2}_{k-1/2}\right) = v^n_k;$$

$$\sqrt{g^n_{k+1/2}} = \frac{1}{3}\,\frac{(R^n_{m+1})^3 - (R_m)^3}{r_{m+1} - r_m}\,\sin\theta_{k+1/2};$$

$$H^n_{k+1/2} = \frac{R_{k+1} - R_k}{R_{k+1} + R_k}\,\frac{2}{\Delta\theta_{k+1/2}};$$

$$\bar{p}^{n+1/2}_{k+1/2} + p\left(\rho^{n+1/2}_{k+1/2}, T^n_{k+1/2}\right) + c\,\rho^n_{k+1/2}\left(v^{n+1/2}_{k+1} - v^{n+1/2}_k\right)^2,$$

$$\text{if} \quad v^{n+1/2}_{k+1} - v^{n+1/2}_k < 0;$$

/80

$$\bar{p}^{n+1/2}_{k+1/2} = p\left(\rho^{n+1/2}_{k+1/2}, T^n_{k+1/2}\right), \ \text{if} \quad v^{n+1/2}_{k+1} - v^{n+1/2}_k \geqslant 0;$$

$$k = 0, 1, \ldots, K;$$

$$v^{n+1/2}_0 = v^{n+1/2}_K = 0; \quad \rho^{n+1/2}_{-1/2} = \rho^{n+1/2}_{1/2}; \quad \rho^{n+1/2}_{K-1/2} = \rho^{n+1/2}_{K+1/2}.$$

Here, for simplicity, the subscript m + ½ is omitted everywhere

and m = 0, 1, ... M - 1.

System (3.1) is solved by the dispersion method for vector values and $v^{n+\frac{1}{2}}$, $\rho^{n+\frac{1}{2}}$ are determined. Then $T^{n+\frac{1}{2}}$ and $U^{n+\frac{1}{2}}$ are found:

$$\varepsilon^{n+1/2}_{k+1/2} + \frac{\tau W^{n+}_{k+1/2}}{\Delta\theta_k}\left(\varepsilon^{n+1/2}_{k+1/2} - \varepsilon^{n+1/2}_{k-1/2}\right) +$$

$$+ \frac{\tau W^{n-}_{k+1/2}}{\Delta\theta_{k+1}}\left(\varepsilon^{n+1/2}_{k\pm 3/2} - \varepsilon^{n+1/2}_{k+1/2}\right) + \frac{\tau\,\bar{p}^{n+1/2}_{k+1/2}}{\rho^{n+1/2}_{k+1/2}\sqrt{g^n_{k+1/2}}\,\Delta\theta_{k+1/2}} \times$$

$$\times\left(\sqrt{g^n_{k+1}}\, v^{n+1/2}_{k+1} - \sqrt{g^n_k}\, v^{n+1/2}_k\right) = \varepsilon^n_{k+1/2} +$$

$$+ \frac{\tau}{\rho^{n+1/2}_{k+1/2}\sqrt{g^n_{k+1/2}}\,\Delta\theta_{k+\frac{1}{2}}}\left\{\varkappa^n_{k+1}\sin\theta_{k+1}\left[-(R_\theta)^n_{k+1}\left(\frac{T^n_{m+1} - T^n_m}{r_{m+1} - r_m}\right)\right]_{k+1} + \qquad (3.2)\right.$$

$$+ (R_r)^n_{k+1} \frac{T^{n+1/2}_{k+3/2} - T^{n+1/2}_{k+1/2}}{\Delta \theta_{k+1}} -$$

$$- \varkappa^n_k \sin \theta_k \left[- (R_\theta)^n_k \left(\frac{T^n_{m+1} - T^n_m}{r_{m+1} - r_m} \right)_k - (R_r)^n_k \frac{T^{n+1/2}_{k+1/2} - T^{n+1/2}_{k-1/2}}{\Delta \theta_k} \right] \Bigg\}.$$

$$(R_\theta)^n_k = \frac{R^n_{k+1/2} - R^n_{k-1/2}}{\Delta \theta_k}; \quad (R_r)^n_k = \frac{R^n_{m+1} - R^n_m}{r_{m+1} - r_m};$$

$$T^{n+1/2}_{-1/2} = T^{n+1/2}_{1/2}, \quad T^{n+1/2}_{K-1/2} = T^{n+1/2}_{K+1/2}; \tag{3.2}$$

$$U^{n+1/2}_{m,k+1/2} = U^n_{m,k+1/2} - \frac{\tau W^{n+}_{m,k+1/2}}{\Delta \theta_k} \left(U^{n+1/2}_{m,k+1/2} - U^{n+1/2}_{m,k-1/2} \right) -$$

$$- \frac{\tau W^{n-}_{m,k+1/2}}{\Delta \theta_{k+1}} \left(U^{n+1/2}_{m,k+3/2} - U^{n+1/2}_{m,k+1/2} \right) + \tau H^n_{m,k+1/2} U^n_{m,k+1/2} v^n_{m,k+1/2} -$$

$$- \tau \left[R^n_{m,k+1/2} \left(\frac{\partial H}{\partial \theta} - H^2 - 1 \right)^n_{m,k+1/2} (v^n_{m,k+1/2})^2 + \right.$$

$$\left. + \tau \frac{H^n_{m,k+1/2}}{\rho^{n+1/2}_{m,k+1/2} R^n_{m,k+1/2}} \cdot \frac{\bar{p}^{n+1/2}_{m,k+1} - \bar{p}^{n+1/2}_{m,k}}{\Delta \theta_{k+1/2}} \right];$$

$$\left(\frac{\partial H}{\partial \theta} \right)^n_{m,k+1/2} = \frac{(H^n_{k+1} - H^n_k)_m}{\Delta \theta_{k+1/2}}.$$

Using $U^{n+\frac{1}{2}}$, $v^{n+\frac{1}{2}}$, $\rho^{n+\frac{1}{2}}$, $T^{n+\frac{1}{2}}$ thus determined, we calculate the second stage, in which the following system of differential equations is solved: /81

$$U^{n+1}_m + \tau \left(R \frac{\partial H}{\partial R} \right)^{n+1/2}_m (Uv)^{n+1/2}_m + \tau \left[\frac{R^2 (1 + H^2)}{q} \right]^{n+1/2}_m \times$$

$$\times \left(\bar{p}^{n+1}_{m+1/2} - \bar{p}^{n+1}_{m-1/2} \right) = U^{n+1/2}_m;$$

$$\frac{1}{\rho^{n+1}_{m+1/2}} = \frac{1}{\rho^{n+1/2}_{m+1/2}} + \frac{\tau}{q^{n+1/2}_{m+1/2}} \left[(R^2 U)^{n+1}_{m+1} - (R^2 U)^{n+1}_m \right];$$

$$R^{n+1}_m = R^n_m + \tau U^{n+1}_m; \quad \left(\frac{\partial H}{\partial R} \right)^{n+1/2}_m = \frac{H^{n+1/2}_{m+1} - H^{n+1/2}_{m-1}}{R^{n+1/2}_{m+1} - R^{n+1/2}_{m-1}};$$

$$\bar{p}^{n+1}_{m+1/2} = p \left(\rho^{n+1}_{m+1/2}, T^{n+1/2}_{m+1/2} \right) + c \rho^{n+1/2}_{m+1/2} \left(U^{n+1}_{m+1} - U^{n+1}_m \right)^2,$$

$$\text{if} \quad U_{m+1} - U_m < 0;$$

$$\bar{p}^{n+1}_{m+1/2} = p \left(\rho^{n+1}_{m+1/2}, T^{n+1/2}_{m+1/2} \right), \text{ если } U_{m+1} - U_m \geqslant 0;$$

$$U^{n+1}_0 = 0; \quad \bar{p}_{m+1/2} = p \left(\theta_{k+1/2}, t^{n+1} \right);$$

$$\varepsilon^{n+1}_{m+1/2} + \tau \frac{\bar{p}^{n+1}_{m+1/2}}{q^{n+1/2}_{m+1/2}} \left[(R^2 U)^{n+1}_{m+1} - (R^2 U)^{n+1}_m \right] =$$

$$= \varepsilon^{n+1/2}_{m+1/2} + \frac{\tau}{q^{n+1/2}_{m+1/2}} \left\{ \varkappa^{n+1/2}_{m+1} \left[(R^{n+1}_{m+1})^2 \frac{\rho^{n+1}_{m+1}}{q^{n+1/2}_{m+1}} (R^2 + R^2_\theta)^{n+1/2}_{m+1} \times \right. \right.$$

$$\times \left(T_{m+3/2}^{n+1} - T_{m+1/2}^{n+1}\right) - (R_\theta)_{m+1}^{n+1/2}\left(\frac{T_{k+1}^{n+1/2} - T_k^{n+1/2}}{\Delta\theta_{k+1/2}}\right)_{m+1}\Bigg] -$$

$$- \varkappa_m^{n+1/2}\Bigg[(R_m^{n+1})^2 \frac{\rho_m^{n+1}}{q_m^{n+1/2}} (R^2 + R_\theta^2)_m^{n+1/2}\left(T_{m+1/2}^{n+1} - T_{m-1/2}^{n+1}\right) -$$

$$- (R_\theta)_m^{n+1/2}\left(\frac{T_{k+1}^{n+1/2} - T_k^{n+1/2}}{\Delta\theta_{k+1/2}}\right)_m\Bigg]\Bigg\}; \quad T_{-1/2}^{n+1} = T_{1/2}^{n+1};$$

$$\left(\frac{\varkappa R}{\sqrt{R^2 + R_\theta^2}}\right)_M^{n+1/2}\Bigg[\rho_M^{n+1}(R^2 + R_\theta^2)_M^{n+1/2}\frac{T_{M+1/2}^{n+1} - T_{M-1/2}^{n+1}}{q_M^{n+1/2}} -$$

$$-\left(\frac{R_\theta}{R^2}\right)_M^{n+1/2}\left(\frac{T_{k+1}^{n+1/2} - T_k^{n+1/2}}{\Delta\theta_{k+1/2}}\right)_M\Bigg] = f\left(T_M^{n+1}\right).$$

Here, for simplicity, we omit the subscript $k + \frac{1}{2}$, $k = 0, 1, \ldots K - 1$.

$$v_{m+1/2,K}^{n+1} = v_{m+1/2,K}^{n} + \frac{\tau}{q_{m+1/2,K}^{n+1/2}}(R^2 H)_{m+1/2,K}^{n+1/2}\left(\bar{p}_{m+1,K} - \bar{p}_{m,K}\right)^{n+1}.$$

§ 4. Systematic calculations, verifying the possibility of calculating by the proposed algorithm, were carried out by the above-described method. Uni-dimensional problems, the solutions of which were known beforehand, were used as examples: a) problem of dispersion of a gaseous sphere in a vacuum [6]; b) problem of shock wave converging on center [6].

The calculations were done in a two-dimensional network, for which a point was selected as the coordinate origin; this point was separated by 1/3 radius from the true center. The initial radius of the sphere in both cases was $R = 1$.

The calculations showed that deviation δ of the outer boundary from the sphere was: /82

in problem a:

θ	0.08	1.34	2.91
σ, %	-2.53	-0.43	2.82

for $R = 2.555$

in problem b:

θ	0.08	1.81	3.06
σ, %	-0.51	0.27	1.1

for R = 0.596

The results were compared with calculations done according to programs that consider spherically symmetric motion. The shape of the shock wave in problem b was nearly spherical, and deviation from the sphere fell within those same limits.

It should be pointed out that the resulting asymmetry was caused not by errors of two-dimensional calculation, but rather by the coarseness of the calculation network in terms of R, since unidimensional calculations, done with networks equivalent to two-dimensional networks on rays θ = 0.08, 1.34, and 2.91, yielded differences comparable to deviations from symmetry in the two-dimensional calculation. Thus, in problem a for $\Delta R = 1/22$ ($\theta = 2.91$) $\delta = 3.69$, for $\Delta R = 1/29$ ($\theta = 1.34$) $\delta = 0$, for $\Delta R = 1/45$ ($\theta = 0.08$) $\delta = 2.59$. The network was uniform with respect to both θ and R. Twenty points were taken for θ.

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